Chapter 3: Theory of Cost and Production Analysis

Production Function:

In economics, a production function relates physical output of a production process to physical inputs or factors of production. It is a mathematical function that relates the maximum amount of output that can be obtained from a given number of inputs - generally capital and labor. The production function, therefore, describes a boundary or frontier representing the limit of output obtainable from each feasible combination of inputs.

Firms use the production function to determine how much output they should produce given the price of a good, and what combination of inputs they should use to produce given the price of capital and labor. When firms are deciding how much to produce they typically find that at high levels of production, their marginal costs begin increasing. This is also known as diminishing returns to scale - increasing the quantity of inputs creates a less-than-proportional increase in the quantity of output. If it weren't for diminishing returns to scale, supply could increasing expand without limits without the price of a good.



Figure: Production function

Iso quant's:

An iso quant (equal quantity) is a curve that shows the combinations of certain inputs such as Labor (L) and Capital (K) that will produce a certain output Q. Mathematically, the data that an iso quant projects is expressed by the equation

$\mathbf{f}(\mathbf{K,L}) = \mathbf{Q}$

This equation basically says that the output that this firm produces is a function of Labor and Capital, where each iso quant represents a fixed output produced with different combinations of inputs. A new iso quant emerges for every level of output.

The Marginal Rate of Technical Substitution (MRTS) equals the absolute value of the slope. The MRTS tells us how much of one input a firm can sacrifice while still maintaining a certain output level. The MRTS is also equal to the ratio of Marginal Productivity of Labor (MP_L): Marginal Productivity of Capital (MP_K). The mathematical form of how Labor (L) can be substituted for Capital (K) in production is given by:

MRTS (L for K)= $-dK/dL = MP_L/MP_K$

Iso costs:

An iso cost line (equal-cost line) is a Total Cost of production line that recognizes all combinations of two resources that a firm can use, given the Total Cost (TC). Moving up or down the line shows the rate at which one input could be substituted for another in the input market. For the case of Labor and Capital, the total cost of production would take on the form:

$\mathbf{TC} = (\mathbf{WL}) + (\mathbf{RK})$

TC= Total Cost, W= Wage, L= Labor, R= Cost of Capital, K= Capital

Example:

A company producing widgets encounters the following costs- cost of capital is \$25000, labor cost is \$15000, and the total cost the firm is willing to pay is \$150,000. Show the iso cost line graphically.

The equation represented by the data is: 150,000 = (15000)L + (25000)K

Setting L=0, we find the y-intercept to be K=6. Setting K=0, we find the x-intercept to be 10

MRTS OR MRS: Marginal Rate of Technical Substitution:

The principle of marginal rate of technical substitution (MRTS or MRS) is based on the production function where two factors can be substituted in variable proportions in such a way as to produce a constant level of output.

Prof. Salvatore defines MRTS thus:

"The marginal rate of technical substitution is the amount of an input that a firm can give up by increasing the amount of the other input by one unit and still remain on the same iso quant."

The marginal rate of technical substitution between two factors K (capital) and L (labour), $MRTS_{IK}$ is the rate at which L can be substituted for K in the production of good X without changing the quantity of output. As we move along an iso quant downward to the right, each point on it represents the substitution of labour for capital.

MRTS is the loss of certain units of capital which will just be compensated for by additional units of labour at that point. In other words, the marginal rate of technical substitution of labour for capital is the slope or gradient of the iso quant at a point. Accordingly, the slope of $MRTS_{Lk} = -\Delta K/\Delta L$. This can be understood with the aid of the iso quant schedule.

Combination	Labour	Capital	$\mathbf{MRTS}_{\mathbf{LK}}(\Delta \mathbf{K}. \Delta \mathbf{L})$	Output
1	5	9		100
2	10	6	3:5	100
3	15	4	2:5	100
4	20	3	1:5	100

The above table shows that in the second combination to keep output constant at 100 units, the reduction of 3 units of capital requires the addition of 5 units of labour, $MRTS_{Lk}$ = 3:5. In the third combination, the loss of 2 units of capital is compensated for by 5 more units of labour, and so on. In Figure 2, at point B, the marginal rate of technical substitution is AS/SB, at point G, it is BT/TG and at H, it is GR/RH.

Law of Substitution or Principle of Least Cost Combination:

The objective of profit maximization can be achieved by two ways, one by increasing output and other by minimizing the cost. The minimization of cost can be possible by deciding the use of more than one resource in substitution of other resources.

The objective of factor-factor relationship is twofold:

- 1) Minimization of cost at a given level of Output.
- 2) Optimization of output to the fixed factors through alternative resource use combinations.

y =f (x1, x2, x3, x4..... xn)

Y is the function of x1 and x2 while other inputs are kept at constant. The relationship can be better explained by the principle of least cost combination.

Principle of Least Cost combination:

A given level of output can be produced using many different combinations of two variable inputs. In choosing between the two completing resources, the saving in the resource replaced must be greater than the cost of resource added. The principle of least cost combination states that if two factor inputs are considered for a given output the least cost combination will be such where their inverse price ratio is equal to their marginal rate of substitution.

1. Marginal Rate of substitution:

MRS is defined as the units of one input factor that can be substituted for a single unit of the other input factor. So MRS of x2 for one unit of x1 is

= Number of unit of replaced resource (x2) Number of unit of added resource (x1)

Price Ratio (PR) =

 $\frac{\text{Cost per unit of added resource}}{\text{Cost per unit of replaced resource}} = \frac{\text{Price of } x1}{\text{Price of } x2}$

Therefore the least cost combination of two inputs can be obtained by equating MRS with inverse price ratio.

i.e. x2 * Px2 = x1 * Px1

This combination can be obtained by following algebraic method or Graphic method.

Iso quant (Iso product) curve:

Iso means equal and quant means quantity. An Iso quant represents the different combinations of two variable inputs used in the production of a given amount of output.

A two-input Cobb-Douglas production function In economics, the Cobb-Douglas functional form of production functions is widely used to represent the relationship of an output to inputs. It was proposed by Knut Wick sell (1851 -1926), and tested against statistical evidence by Charles Cobb and Paul Douglas in 1928. In 1928 Charles Cobb and Paul Douglas published a study in which they modeled the growth of the American economy during the period 1899 - 1922. They considered a simplified view of the economy in which production output is determined by the amount of labor involved and the amount of capital invested. While there are many other factors affecting economic perform, their model proved to be remarkably accurate.

The function they used to model production was of the form: P (L, K) = $bL\alpha K\beta$ where:

- P = total production (the monetary value of all goods produced in a year)
- L = labor input (the total number of person-hours worked in a year)

• K = capital input (the monetary worth of all machinery, equipment, and buildings)

• b = total factor productivity

 α and β are the output elasticity's of labor and capital, respectively. These values are constants determined by available technology.

Output elasticity measures the responsiveness of output to a change in levels of either labor or capital used in production, ceteris paribus. For example if $\alpha = 0.15$, a 1% increase in labor would lead to approximately a 0.15% increase in output. Further, if: $\alpha + \beta = 1$, the production function has constant returns to scale. That is, if L and K are each increased by 20%, then P increases by 20%.

Returns to scale refers to a technical property of production that examines changes in output subsequent to a proportional change in all inputs (where all inputs increase by a constant factor). If output increases by that same proportion, then there are constant returns to scale (CRTS) sometimes referred to simply as returns to scale. If output increases by less than that proportional change, there are decreasing returns to scale (DRS). If output increases by more than that proportion, there are increasing returns to scale (IRS).

However, if $\alpha + \beta < 1$, returns to scale are decreasing, and if $\alpha + \beta > 1$, returns to scale are increasing. Assuming perfect competition, α and β can be shown to be labor and capital's share of output.



FIGURE1: Cobb-Douglas Production function.

Laws of Returns:

The laws of returns to scale can also be explained in terms of the iso quant approach. The laws of returns to scale refer to the effects of a change in the scale of factors (inputs) upon output in the long run when the combinations of factors are changed in the same proportion.

If by increasing two factors, say labour and capital, in the same proportion, output increases in exactly the same proportion, there are constant returns to scale. If in order to secure equal increases in output, both factors are increased in larger proportionate units, there are decreasing returns to scale. If in order to get equal increases in output, both factors are increased in smaller proportionate units, there are increasing returns to scale.

The returns to scale can be shown diagrammatically on an expansion path "by the distance between successive 'multiple-level-of-output" iso quant's', that is, iso quant's that show levels of output which are multiples of some base level of output, e.g., 100, 200, 300, etc."

Increasing Returns to Scale:

Shows the case of increasing returns to scale where to get equal increases in output, lesser proportionate increases in factors, labour and capital, are required.



It follows that in the figure:

- 100 units of output require 3C + 3L
- 200 units of output require 5C + 5L
- 300 units of output require 6C + 6L

So that along the expansion path OR, OA > AB > BC. In this case, the production function is homogeneous of degree greater than one. The increasing returns to scale are attributed to the existence of indivisibilities in machines, management, labour, finance, etc. Some items of equipment or some activities have a minimum size and cannot be divided into smaller units. As the business unit

expands, the returns to scale increase because the indivisible factors are employed to their full capacity.

Increasing returns to scale also result from specialization and division of labour. When the scale of the firm expands there is wide scope for specialization and division of labour. Work can be divided into small tasks and workers can be concentrated to narrower range of processes. For this, specialized equipment can be installed.

Internal economies of scale (IEOS):

Expensive capital inputs:

Large-scale businesses can afford to invest in expensive and specialist capital machinery. For example, a supermarket might invest in new database technology that improves stock control and reduces transportation and distribution costs. It may not be viable for a small corner shop to buy this technology. We find that highly expensive fixed units of capital are common in every mass manufacturing production process.

Specialization of the workforce:

Within larger firms there is the possibility of splitting production processes into separate tasks to boost productivity. The use of division of labour in the mass production of motor vehicles and in manufacturing electronic products is an example of this type of technical economy of scale.

The law of increased dimensions (or the "container principle".):

This is linked to the cubic law where doubling the height and width of a tanker or building leads to a more than proportionate increase in the cubic capacity – the application of this law opens up the possibility of scale economies in distribution and freight industries and also in travel and leisure sectors with the emergence of super-cruisers such as P&O's Ventura. Consider the new generation of super-tankers and the development of enormous passenger aircraft capable of carrying well over 500 passengers on long haul flights. The law of increased dimensions is also important in the energy sectors and in industries such as office rental and warehousing.

Learning by doing:

There is growing evidence that industries learn-by-doing! The average costs of production decline in real terms as a result of production experience as businesses cut waste and find the most productive means of producing output on a bigger scale. Evidence across a wide range of industries into so-called "progress ratios", or **"experience curves" or "learning curve effects":**

Indicate that unit manufacturing costs typically fall by between 70% and 90% with each doubling of cumulative output. Businesses that expand their scale can achieve significant learning economies of scale.



Monophony power:

A large firm can purchase its factor inputs in bulk at discounted prices if it has monophony (buying) power in the market. A good example would be the ability of the electricity generators to negotiate lower prices when finalizing coal and gas supply contracts. The national food retailers also have significant monophony power when purchasing supplies from farmers and wine growers and in completing supply contracts from food processing businesses. Other controversial examples of the use of monophony power include the prices paid by coffee roasters and other middle men to coffee producers in some of the poorest parts of the world.

Managerial economies of scale:

This is a form of division of labour where firms can employ specialists to supervise production systems. Better management; increased investment in human resources and the use of specialist equipment, such as networked computers can improve communication, raise productivity and thereby reduce unit costs.

Financial economies of scale:

Larger firms are usually rated by the financial markets to be more 'credit worthy' and have access to credit with favorable rates of borrowing. In contrast, smaller firms often pay higher rates of interest on overdrafts and loans. Businesses quoted on the stock market can normally raise fresh money (extra financial capital) more cheaply through the sale (issue) of equities to the capital market. They are also likely to pay a lower rate of interest when they issue bonds because of a better credit rating.

Network economies of scale:

This type of economy of scale is linked more to the growth of demand for a product – but it is still worth understanding and applying.) There is growing interest in the concept of a network economy. Some networks and services have huge potential for economies of scale. That is, as they are more widely used (or adopted), they become more valuable to the business that provides them. We can identify networks economies in areas such as online auctions and air transport networks. The marginal cost of adding one more user to the network is close to zero, but the resulting financial benefits may be huge because each new user to the network can then interact, trade with all of the existing members or parts of the network. The rapid expansion of e-commerce is a great example of the exploitation of network economies of scale. EBay is a classic example of exploiting network economies of scale as part of its operations.

Cost Analysis:

Cost analyses of social service programs can be usefully conducted at two different points in time: prior to implementing a new program and after a program is already running (even if the program has been in operation for a number of years). As discussed next, each type of analysis can serve multiple purposes, although they differ in important respects.

Cost analyses of social programs are conducted much more easily once a program has been implemented. Cost analyses of not-yet-implemented programs must typically be based on projections or informed guesses about such issues as the number of people who will actually receive services and the length of time participants will receive those services. Hence, they are inherently subject to considerably more uncertainty than analyses of ongoing programs, for which much information is already available.

Opportunity cost:

This concept of scarcity leads to the idea of opportunity cost. The opportunity cost of an action is what you must give up when you make that choice. Another way to say this is: it is the value of the next best opportunity. Opportunity cost is a direct implication of scarcity. People have to choose between different alternatives when deciding how to spend their money and their time. Milton Friedman, who won the Nobel Prize for Economics, is fond of saying "there is no such thing as a free lunch." What that means is that in a world of scarcity, everything has an opportunity cost. There is always a trade-off involved in any decision you make.

The concept of opportunity cost is one of the most important ideas in economics. Consider the question, "How much does it cost to go to college for a year?" We could add up the direct costs like tuition, books, school supplies, etc. These are examples of explicit costs, i.e., costs that require a money payment. However, these costs are small compared to the value of the time it takes to attend class, do homework, etc. The amount that the student could have earned if she had worked rather than attended school is the implicit cost of attending college. Implicit costs are costs that do not require a money payment. The opportunity cost includes both explicit and implicit costs. Explicit costs are costs that require a money payment. Implicit costs are costs that do not require a money payment.

Opportunity cost includes both explicit and implicit costs. The notion of opportunity cost helps explain why star athletes often do not graduate from college. The cost of going to school includes the millions of dollars they could earn as professional athletes. If Kobe Bryant had decided to attend college for four years after high school instead of signing with the Lakers, his implicit cost would have been over \$10 million, the salary he earned in his first four years as Lakers.

Fixed vs. Variable costs, Explicit costs Vs. Implicit costs:

Fixed Costs:

The definition of a fixed cost is one which does not vary in total when the level of output by the business does vary. In other words, when the Sales level within a business increases then the fixed costs in total would not increase. It also follows that when the Sales level in a business decreases, the fixed costs would not decrease.

An example of a fixed cost for a business making a product such as a bakery would be the business rates. For a business producing a service such as massage therapy would be any costs associated with the rent or ownership of premises, insurance, and costs associated with the ownership of equipment. As fixed costs are not dependent upon the level of output (sales), they are often expressed as being per period of time, for example annually, weekly or monthly. It can be helpful to picture costs in the form of a chart.



VARIABLE COSTS:

Variable costs are those which DO vary as a total cost to the organization when output (number of items or services produced) varies. In fact a true variable cost will vary in exactly the same proportion as the output. In other words, as sales increase the variable costs increase. An example of a variable cost for a bakery would be the cost of flour. In a service business, there are often fewer variable costs. Often the main variable cost in providing a service is the cost of wages for an employee working directly in providing the service. Other variable costs in a service business would be anything directly 'used up' during the provision of the service. For example with massage therapy, oil may be used and there may be the cost of laundering one or two towels. Variable costs should be able to be expressed per item of output or sales. If this proves to be very difficult, you may need to classify such costs as fixed.



NOTE: The following data relating to employee. This is very useful while we are studying the fixed and variable costs in economics subject.

What about Wages/Labour?

If you are employing someone, this is obviously a cost to the business. Whether it is fixed or variable will depend on the nature of their terms of employment. Consider the following scenarios; Employee paid a basic wage for 37hours per week

Fixed or variable cost?

Employee paid a set amount per item produced (e.g. per therapy treatment or per floral wreath). Employee paid for number of hours worked but no minimum hours guaranteed (e.g. housekeeping staff paid to service holiday accommodation only when let)

What about Your Own 'Wages'/Labour?

If you are a sole trader or trading as a partnership, your own earnings are not a cost, they are a withdrawal from the profit, and are called 'drawings'. However, when looking at variable costs in order to price your product or service, you should include a realistic figure for any direct labour involved to make sure that you are costing and pricing your product realistically. Your price to the customer must obviously be higher than the variable cost of producing it!

Explicit costs Vs. Implicit costs:

Explicit costs:

Are expenses for which one must pay with cash or equivalent? Because a cash transaction is involved, they are relatively easily accounted for in analysis. These costs are never hidden, one has to pay separately. **Example**: Electricity Bill, wages to workers etc.

Implicit costs:

Do not involve a cash transaction, and so we use the opportunity cost concept to measure them. Implicit costs are related to forgone benefits of any single transaction. These are intangible costs that are not easily accounted for. **Example**: The time and effort that an owner puts into the maintenance of the company rather than working on expansion.



- These three concept diagrams make clear the meanings of total revenue, economic profit, accounting profit, explicit costs and implicit costs.
- Total Revenue is all the money the firm brings in.
- Accounting Profit is the money left over after explicit costs are deducted.
- Economic Profit is what's left after the Explicit and Implicit costs are deducted.
- Normal Profit is when the accounting profit = implicit costs (hence no economic profit).
- If the firm is selling at less than the average variable cost then it is making an economic loss and should shut down temporarily. (Incurring a loss of the Total Fixed Cost - i.e. plant costs etc).
- If the firm is selling at greater than the minimum average variable cost then it produces such that it is making normal profit (MR = MC).
- If the firm is selling at greater than the average variable cost then it is in economic profit.

Out of pocket costs vs. Imputed costs:

Out-of-Pocket Costs:

Out-of-pocket costs are actual expenses you incur. If you buy land for \$100,000 and lose the opportunity to buy discounted machinery you'll need to buy later at a price that will be \$50,000 higher, your out-of-pocket costs on the land purchase is rs.100000. Out-of-pocket costs do not include any future repairs, improvements, depreciation, lawsuits or other expenses associated with your purchase. If you buy something on credit, the interest on the money you borrowed to make the purchase can be considered an out-of-pocket cost over the life of the loan.

Imputed cost:

Imputed cost of capital or opportunity cost is the benefit foregone by investing the money in business.

Example:

If the risk free rate of return of a government bond is5 %, then5% is the imputed cost of capital. Cost of capital is the expectation of ht e shareholders or owner from the company. It is usually calculated for a joint stock form of ownership. It is the sum of cost of debenture & amp; cost of equity. Cost of debenture is the after tax interest rate. Cost of equity is the dividend rate.



If the output of butter increases by 10 units then there will be an opportunity cost of 10 guns. A straight-line production possibility frontier will have a constant opportunity cost as can be seen here. For any increase in butter production of 10 there is an opportunity cost of 10 guns.

Break-even Analysis (BEA)-Determination of Break-Even Point:

A person starting a new business often asks, "At what level of sales will my company make a profit?" Established companies that have suffered through some rough years might have a similar question. Others ask, "At what point will I be able to draw a fair salary from my company?" Our discussion of break-even point and break-even analysis will provide a thought process that may help to answer those questions and to provide some insight as to how profits change as sales increase or decrease.

Frankly, predicting a precise amount of sales or profits is nearly impossible due to a company's many products (with varying degrees of profitability), the company's many customers (with varying demands for service), and the interaction between price, promotion and the number of units sold. These and other factors will complicate the break-even analysis.

In spite of these real-world complexities, we will present a simple model or technique referred to by several names: break-even point, break-even analysis, break-even formula, break-even point formula, break-even model, cost-volume-profit (CVP) analysis, or expense-volume-profit (EVP) analysis. The latter two names are appealing because the break-even technique can be adapted to determine the sales needed to attain a specified amount of profits. However, we will use the terms break-even point and break-even analysis.

To assist with our explanations, we will use a fictional company Oil Change Co. (a company that provides oil changes for automobiles). The amounts and assumptions used in Oil Change Co. are also fictional. The basic equation for determining the break-even units is:

The basic equation for determining the break-even sales:



In the diagram given above, the break- even point is reached where TR equals TC. Prior to this stage, the TC is above TR and the firm is making losses. It starts earning profits after the break- even point.

Break- Even analysis is a concept used widely in the production management and costing. It is an analytical tool which helps the firm to identify that level of sale where it will cover its cost of production. Any sale over and above the break- Even Point will accrue profits to the firm, while any sales less than it would put the firm into losses. The Break- Even Point shows the price at which the firm makes neither profit nor loss.

Symbolically, in the equation form, the break-even point (in terms of Unit Sales (X)) can be directly computed in terms of Total Revenue (TR) and Total Costs (TC) as:

TR = TC

 $\mathbf{P} \times \mathbf{X} = \mathbf{TFC} + \mathbf{V} \mathbf{x} \mathbf{X}$

 $(\mathbf{P} \times \mathbf{X})$ - $(\mathbf{V} \times \mathbf{X}) = \mathbf{TFC}$

 $(\mathbf{P} - \mathbf{V}) \times \mathbf{X} = \mathbf{TFC}$

$$X = \frac{TFC}{(P-V)}$$

Where,

- TFC is Total Fixed Costs,
- P is Unit Sale Price,
- V is Unit Variable Cost.

In other words, if we multiply Unit Sales (X) with the Unit Sale Price (P) we will get the sales revenue amount. The break-even sales can also be measured by dividing fixed costs by Contribution Margin Ratio, i. e.

$$X = \frac{TFC}{C}$$

Where, C is Contribution Margin Ratio. The Contribution Margin Ratio shows the contribution towards the recovery of fixed operating costs, i. e.

$$X = \frac{(\mathbf{p} - \mathbf{v})}{c} \times 100$$

NOTE: The below problem will explain the break even analysis

Sales		
Sales price per unit	25.00	
Sales Volume (Units)	1,000.00	
Total Sales		25,000.00
Variable Costs Per Unit		
Commission	3.50	
Direct Material	4.00	
Shipping	1.50	
Supplies	1.20	
Packaging	3.00	
Variable Costs Per Unit	13.20	
Total Variable Costs		13,200.00
Unit Contribution Margin	27.22	
Gross Margin		11,800.00
Fixed Costs		
Administrative	1,350.00	
Insurance	650.00	
Property Tax	1,250.00	
Rent	1,000.00	
Depreciation	1,350.00	
Total Fixed Costs		5,600.00
Net Profit (or Loss)	6,200.00	
Breakeven Point (Units Sold)		475.00

Managerial Significance and limitations of BEA:

The prime function of a management executive in a business organization indecision and forward planning. Decision Making means the process of selecting one action from two or more alternative courses of action whereas forward planning means establishing plans for the future. The question of choice arises because resources such as capital, land, labour and management are limited and can be employed in alternative uses. The decision making function thus becomes one of making choices or decisions that will provide the most efficient means of attaining a desired end, say, profit maximization. Once decision is made about the particular goal to be achieved, plans as to production, pricing, capital, raw materials, labour, etc., are prepared. Forward planning thus goes hand in hand with decision making.

A significant characteristic of the conditions, in which business organizations work and take decisions, is uncertainty. And this fact of uncertainty not only makes the function of decision making and forward planning complicated but adds a different dimension to it. If knowledge of the future were perfect, plans could be formulated without error and hence without any need for subsequent revision. In the real world, however, the business manager rarely has complete information and the estimates about future predicted as best as possible. As plans are implemented over time, more facts become known so that in their light, plans may have to be revised, and a different course of action adopted. Managers are thus engaged in a continuous process of decision making through an uncertain future and the overall problem confronting them is one of adjusting to uncertainty.

Limitations of Break-Even Analysis:

Although, break-even analysis is a very useful risk assessment technique and a useful device for testing the sensitivities of business performance, the following limitations must be considered:

- All costs resolved into fixed or variable
- Variable costs fluctuate in direct proportion to volume.
- Fixed costs remain constant over the volume range.
- The selling price per unit is constant over the entire volume range.
- The company sells only one product, or mix of products tends to remain constant.
- Volumetric increase is the only factor affecting costs.
- The efficiency in the use of resources will remain constant over the period.